Paper Reference(s)
6682/01

Edexcel GCE

Mechanics M6

Advanced/Advanced Subsidiary

Tuesday 31 January 2006 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question paper
Answer book (AB16) Graph paper (ASG2) Mathematical Formulae and Statistical Tables (Lilac or Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M6), the paper reference (6682), your surname, other names and signature.
Check that you have the correct question paper.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 6 questions in this question paper. The total mark for this paper is 75.
There are 8 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the examiner. Answers without working may gain no credit.

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1. Relative to a fixed pole O and an initial line, the curve C has polar equation $r = a \sin 2\theta$, where a is a positive constant. A particle P moves along C so that OP rotates with constant angular speed ω.

(a) Find, in terms of a and ω, the speed of P when $\theta = \frac{\pi}{8}$.

(5)

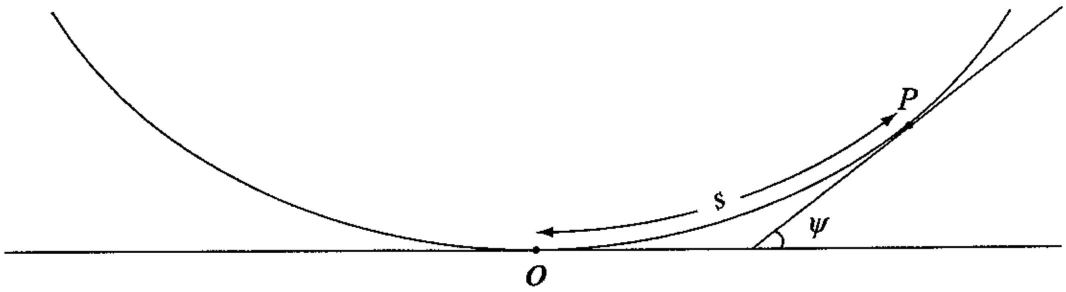
(b) Show that, when $\theta = \frac{\pi}{4}$, the transverse component of the acceleration of P is zero.

(3)

(Total 8 marks)

2.

Figure 1



A smooth wire is in the shape of an arc of the cycloid $s = 2a\sin\psi, -\frac{\pi}{2} < \psi < \frac{\pi}{2}$.

The wire is fixed in a vertical plane with its vertex, the origin O, at its lowest point. A bead of mass m is threaded on the wire. At time t the bead is at the point P, the arc length $OP = s$, and the tangent at P makes an angle ψ with the horizontal, as shown in Figure 1. The bead is released from rest at the point where $s = \frac{3}{2}a$.

(a) Show that $\ddot{s} = -\left(\frac{g}{2a}\right)s$.

(2)

(b) Show that the speed of the bead at O is $\frac{3}{2}\sqrt{\left(\frac{ag}{2}\right)}$.

(3)

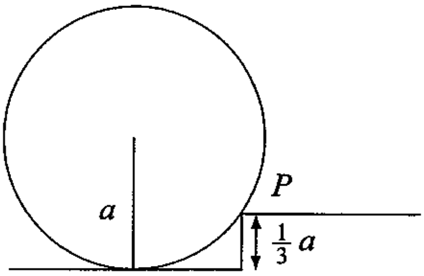
(c) Find the magnitude of the normal reaction exerted on the bead by the wire as the bead passes through O.

(5)

(Total 10 marks)

3.

Figure 2



A uniform hoop, of mass m and radius a, rolls without slipping with its plane vertical along a horizontal path until it reaches a vertical step of height $\frac{1}{3}a$, as shown in Figure 2. The step is at right angles to the path. When the hoop reaches the step at P, it starts to turn about P, without slipping or losing contact. The angular speed of the hoop immediately before it makes contact with P is ω.

Find the angular speed of the hoop immediately after making contact with P.

(Total 9 marks)

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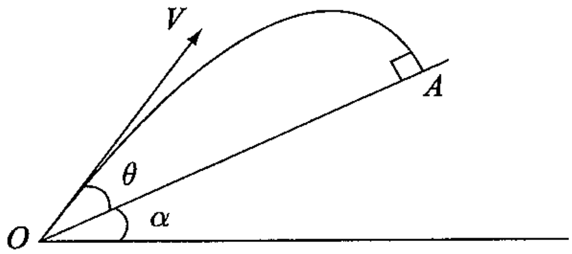
4. A uniform solid sphere, of mass m and radius a, is projected down a line of greatest slope of a rough plane inclined at an angle α to the horizontal. The coefficient of friction between the sphere and the plane is μ. Initially the sphere has no angular speed and the speed of the sphere down the plane is V. Given that $\mu > \frac{2}{7}\tan\alpha$, show that the sphere will slip for a time

$$\frac{2V}{g(7\mu\cos\alpha - 2\sin\alpha)}.$$

(Total 15 marks)

5.

Figure 3



A ball is projected from the point O at the bottom of a hill to hit a target A from above. The ball is modelled as a particle, the target by a point, and the hill as a plane inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$. The particle travels in a vertical plane through a line of greatest slope of the inclined plane. The particle hits the plane at right angles at A, as shown in Figure 3. The particle is projected with initial speed V m s^{-1} at an angle θ to the plane. By considering the motion along and perpendicular to the plane, or otherwise,

(a) show that $\tan\theta = \dfrac{2}{3}$.

(8)

Given that the particle takes 2 s to move from O to A,

(b) find the distance OA,

(5)

(c) find the value of V.

(3)

(Total 16 marks)

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6. A particle P of mass 2 kg is free to move on a smooth horizontal table. The particle is atta to one end of a light elastic string of natural length 1 m and modulus of elasticity 4 N. The oth end of the string is attached to a fixed point O on the table. The position of P is specified by polar coordinates (r, θ) referred to O as pole and a fixed line OA as initial line, where $OP = r$ metres.

Initially P is held at a point on OA with the string just taut. It is projected horizontally at right angles to OA with a speed of 3 m s^{-1}. Given that the string remains taut, show that, t seconds after projection,

(a)

$$\frac{\mathrm{d}^2r}{\mathrm{d}t^2} = \frac{9}{r^3} - 2(r-1),$$

(8)

(b)

$$\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 = 7 - \frac{9}{r^2} - 2r^2 + 4r.$$

(4)

(c) Hence show that, in the subsequent motion, the length of the string varies between 1 m and 3 m.

(5)

(Total 17 marks)

TOTAL FOR PAPER: 75 MARKS

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